Appropriate Mathematical Understanding for Effective Teaching

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Lakehead UNIVERSITY

faculty of education

Overview

- 5 year study of elementary and early secondary teachers, both preservice (over 500) and in-service (about 100)
- Examined beliefs about mathematics as well as knowledge, and how these developed during professional development
- Quantitative and qualitative study (statistically analysed written survey, semi-structured interviews, classroom observations, professional learning group observations, focus groups etc)

My own viewpoint … math or education?

Previously …

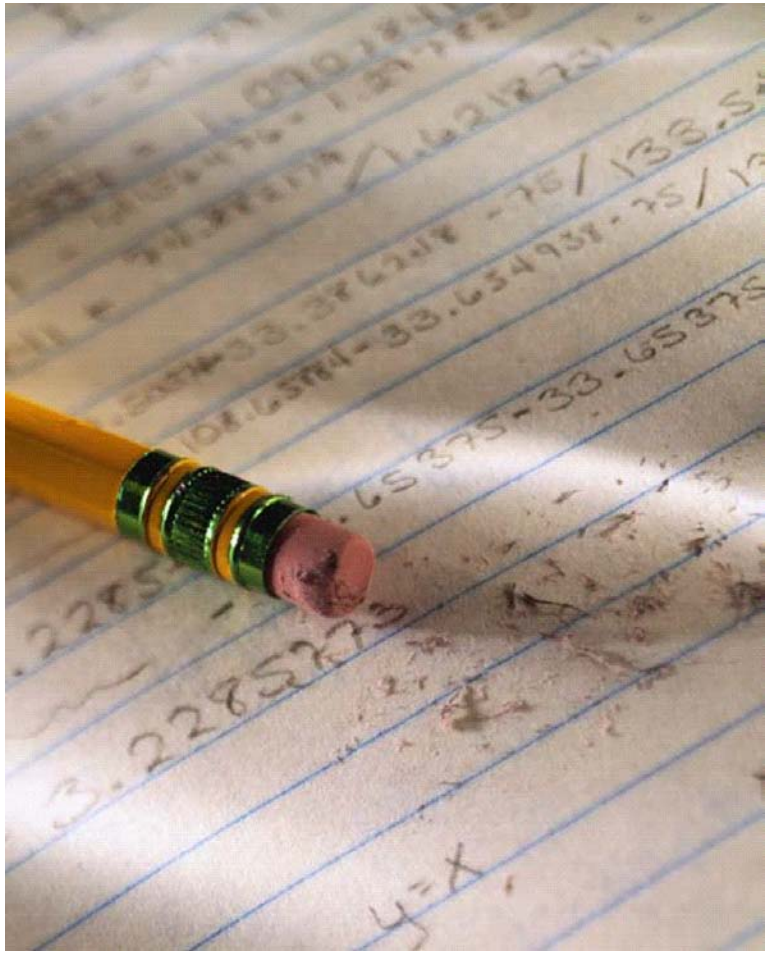
- Undergraduate mathematics degree
- Graduate mathematics education background
- Contract lecturer in the Mathematics Department
- Classroom teacher

Currently …

- Mathematics educator in the Faculty of Education

Mathematics Needs of Teachers: Are they ….

- remedial?
- specialised?
- both?



Math is the “deal-breaker” for reform:

- For example, Wong & Lai (2006) found that mathematics knowledge as needed for teaching “is the crucial factor leading to effective mathematics teaching” (p.1)
- A distinct body of knowledge (Davis & Simmt, 2006)

“Mathematics for Teaching”

- Specialised knowledge not needed in other settings (Ball, Thames & Phelps, 2008)
- It is “qualitatively different” (Davis & Simmt, 2006, p. 294) than the knowledge expected of students
- Not statistically related to subject-content knowledge (Wong & Lai, 2006)
- Tends to “fall through the cracks” in teacher education (Kajander, in press)

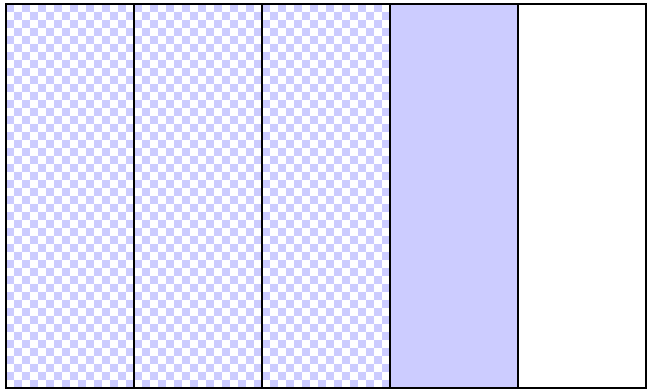
An Example …

- The paper pieces represent one whole cut into fifths
- Take out four of those fifths and:

A. Show ¾ of the 4/5

Conceptual Knowledge

- Starting with 4/5 we see that ¾ of it is 3/5

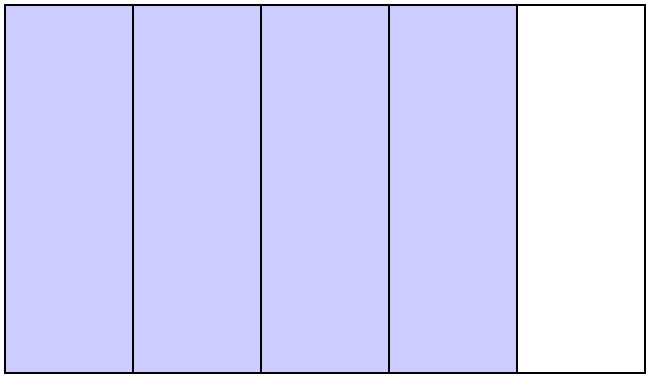


Procedural Knowledge

$$\frac{3}{4} \times \frac{4}{5} = \frac{3 \times 4}{4 \times 5} = \frac{12}{20} = \frac{12 \div 4}{20 \div 4} = \frac{3}{5}$$

Another example with the fifths

- Start with your four fifths model again
- Now show 2/3 of the 4/5



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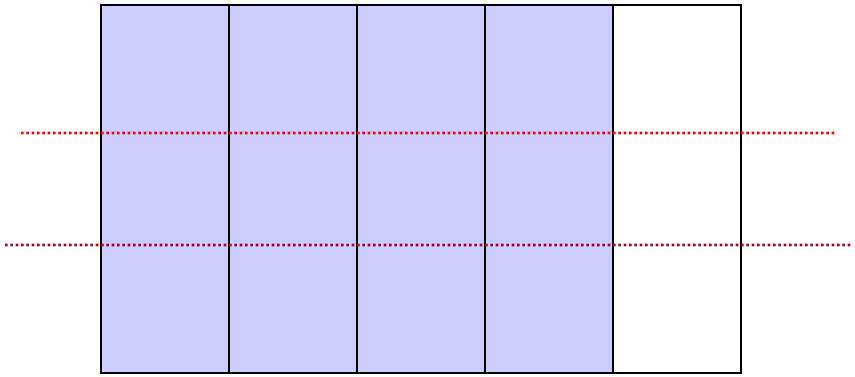

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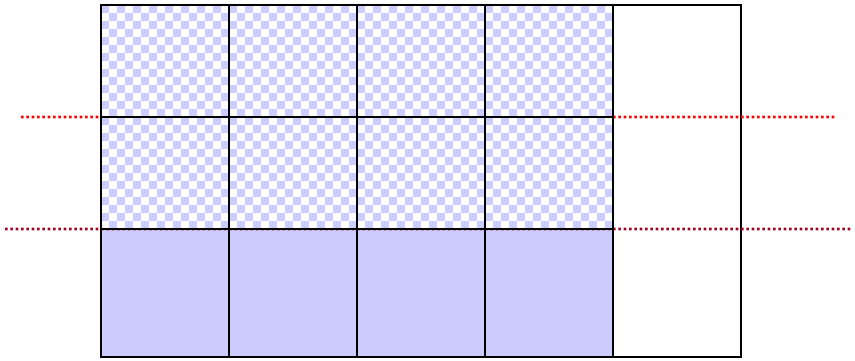

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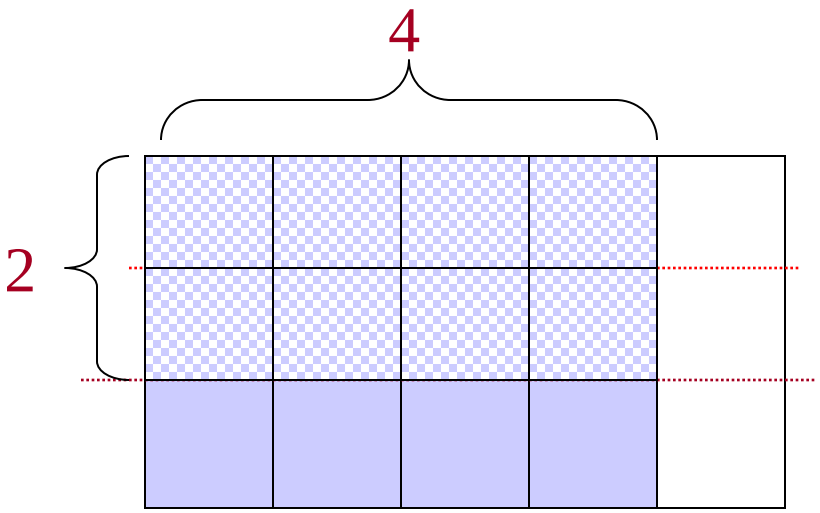

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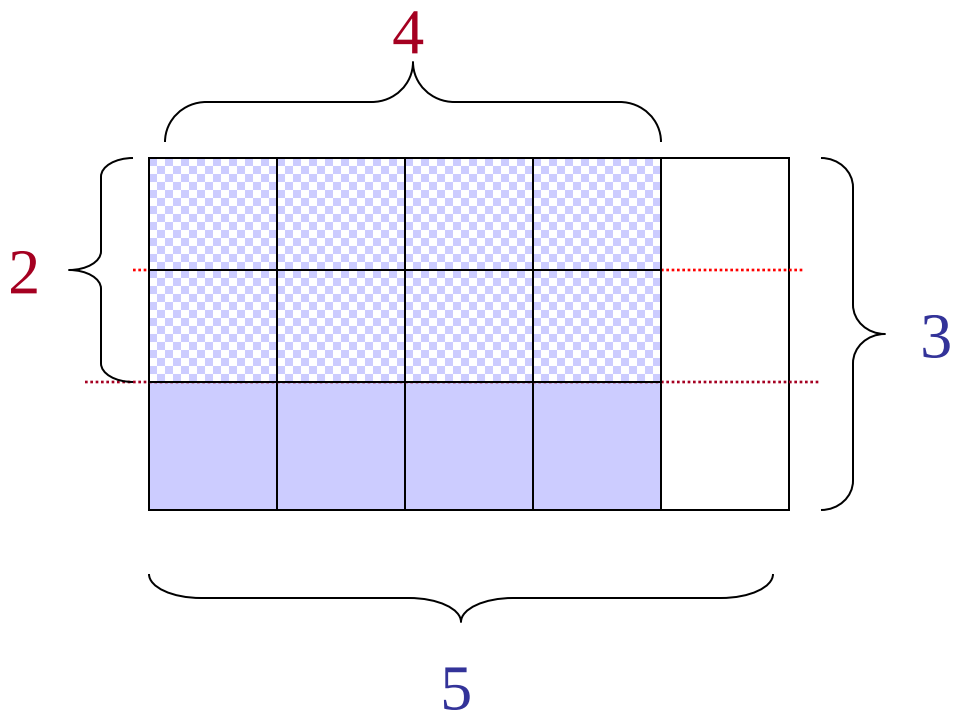

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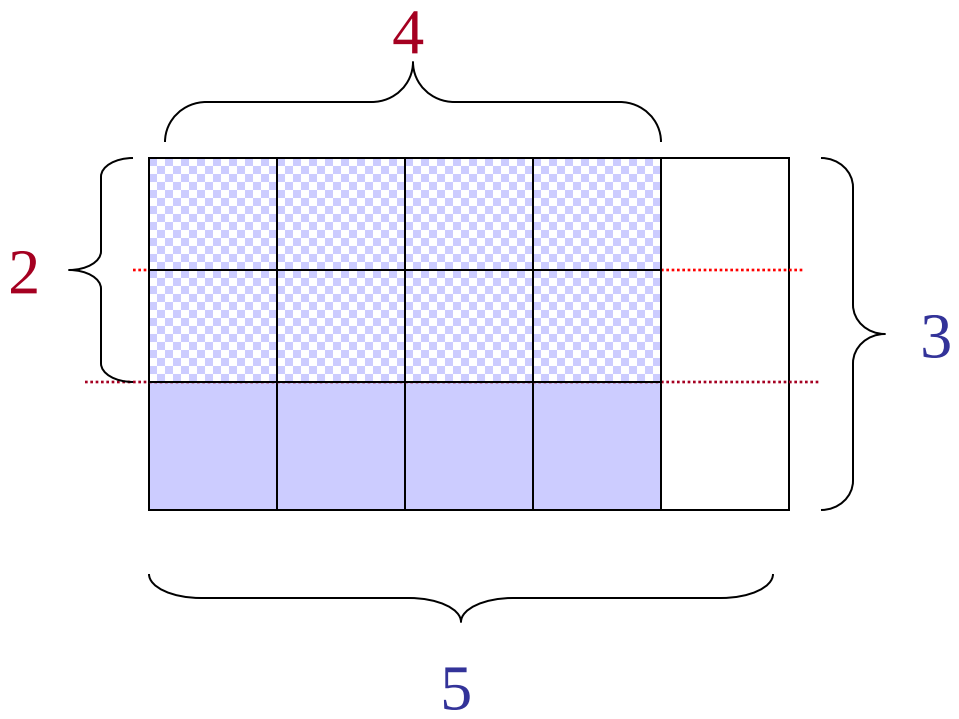


$$\frac{2 \times 4}{3 \times 5}$$

What if …?

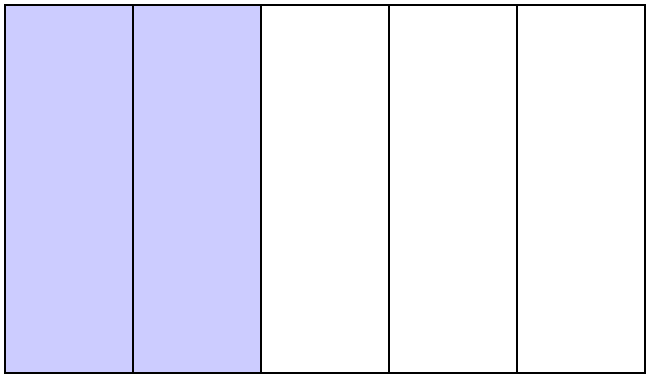
- What if we asked for a model to solve

$$\frac{3}{4} \times \frac{2}{5} \quad ?$$

Would we have managed (necessarily) to illustrate the standard procedure?

¾ of 2/5

- Start with a two fifths model
- Now show 3/4 of the 2/5



¾ of 2/5

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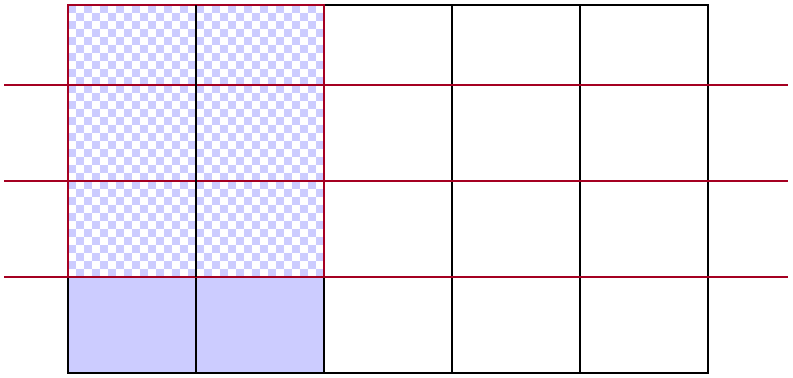
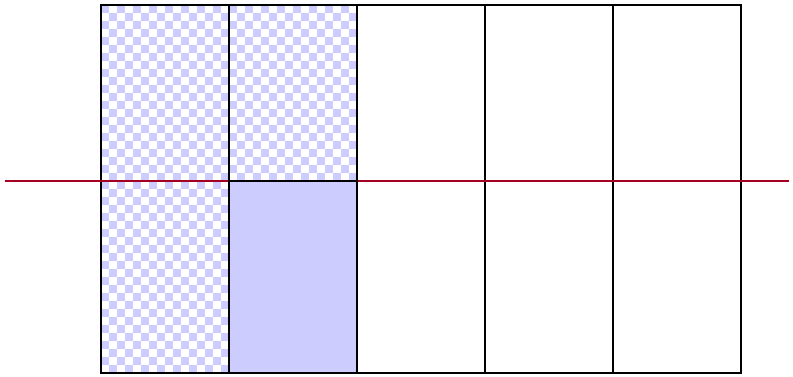


We want students to find the answer as an area that is 3 x 2, out of a total area of 4 x 5, in order to “invent” the standard procedure

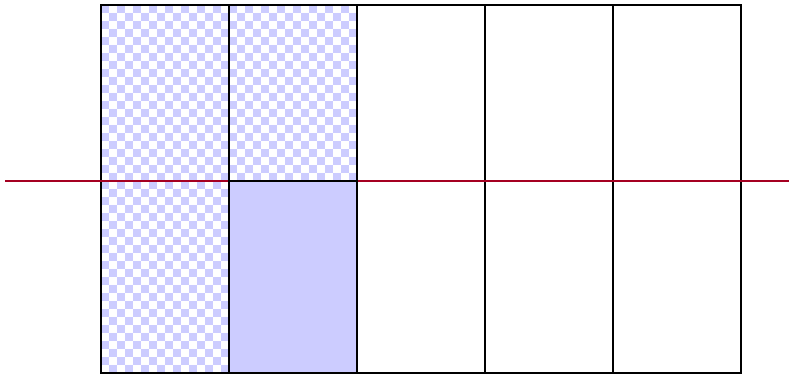
¾ of 2/5

- What about this way?



¾ of 2/5

- Oops …. The model can be used to do this *without* cutting it up like the standard procedure (Idea: ¾ of 4/10 is 3/10)



“Math for Teaching”

- As well as deeply understanding the models, a teacher would have to know *which* models would give the best chance of students actually deriving the mathematical ideas intended in a lesson

Summary:

- Procedural knowledge
- Conceptual knowledge
- Mathematics for Teaching

This study focused on the conceptual knowledge which underpins mathematics for teaching

What's "different" from content knowledge?

- *Mathematics for Teaching* is a kind of specialised conceptual knowledge that allows teachers to help students necessarily develop new concepts from previously learned (and relatively more concrete) ideas.
- (Other aspects include knowledge of students' typical errors and how to identify the mathematical elements of these, and so on)

According to the literature

- Mathematics educators have argued for some time for the need for such a “specialised” study of mathematics for teachers (eg. Davis and Simmt, 2003; 2006; Ball, Hill, and Bass, 2005; Ball, Thames, and Phelps, 2008; Silverman and Thompson, 2008; Kajander, 2007) in which the mathematics contains ideas generally not included in standard undergraduate mathematics courses

How much time ?

- National Council on Teacher Quality (2008)
- Working Group on Elementary Mathematics for Teaching (CMEF 2009)
- Lakehead University BEd candidates (none to potentially 72 hours upon entry; potential for 12 to 32 more hours during BEd year)

The Study – Both preservice and in-service teachers

The Study

- Five year study of junior intermediate (grades 4 to 10) preservice teachers in their teacher certification year (N > 500), as well as in-service teachers (about 100).
- Quantitative data (pre/post survey)
- Qualitative data (semi-structured interviews, samples of their work, journals and emails)

The instrument

- Survey has been created and statistically validated using a well-known large-scale instrument (Hill et al, 2004) for assessing teachers' content knowledge (Kajander, 2007; Zerpa, 2008; Zerpa, Kajander & van Barneveld, 2009)

The instrument

- Mathematics items separated into sub-categories:

Procedural Knowledge (PK):

eg. ***Calculate*** *1 ¾ ÷ ½ showing your steps*

Conceptual Knowledge (CK):

eg. ***Explain*** *why and how the method you used works, using explanations, diagrams, models, and examples as appropriate*

1. Results – Preservice teachers

- Pre and post-test scores for 4 years (N=426)
- Pretest scores for 5 years (N= 585)

Results to date for the following research questions:

- Does high school and university mathematics background make a difference in teacher candidates' initial conceptual knowledge (CK) as they enter a mathematics methods course?
- What university mathematics courses make the most difference? (What is 'enough' preparation?)
- What levels of conceptual understanding of mathematics as needed for teaching are demonstrated by teacher candidates upon entry to the teacher certification program?

Results - High school background

- Candidates with more and higher level mathematics courses in high school arrived at methods courses significantly better prepared in terms of conceptual understanding (CK).
- For example, people with grade 11 advanced or university level courses were generally stronger that those with grade 12 general or non “U” level courses

University background

- Candidates with a math or science-related undergraduate degree (N=97) vs. other degree (N=486) were significantly stronger procedurally and conceptually at the beginning of the methods course and remained so at the end

BUT

- None of the subgroups (including those with a ‘math’-related degree) demonstrated strong or even adequate conceptual understanding of grade 4 to 10 mathematics at the start of the teacher certification program (Initial conceptual knowledge means of each cohort are consistently about 10 to 20 %).

Levels of knowledge by undergraduate majors entering BEd

	procedural knowledge	conceptual knowledge
Math, Engineering, Science Majors	82%	21%
Other Majors	69%	11%

Types of undergraduate math courses

- Pretest mean scores in CK; pretest total N = 585, overall mean 12.5%, (descriptive statistics only):

1. no university math courses (9%); N=267

2. undergraduate math course for teachers only (12%); N=74

3. at least one regular math course but not teachers' course (16%); N=232

4. teachers' course plus one or more other undergrad math (22%); N = 12

Issue with appropriate content

- MOST students taking undergrad math course for teachers are primary junior (for teaching up to grade 6) and many have a very weak high school background
- Is there a need for a separate section or a new course focused more on the content related to the intermediate level which would better support both J/I (gr 4-10) and I/S (gr 7-12) teacher candidates?

Types of undergraduate math courses

- Post-test mean scores in CK; post-test total N = 426, overall mean 54.5% (descriptive statistics only):

1. no university math courses (49%)

2. undergraduate math course for teachers only (51%)

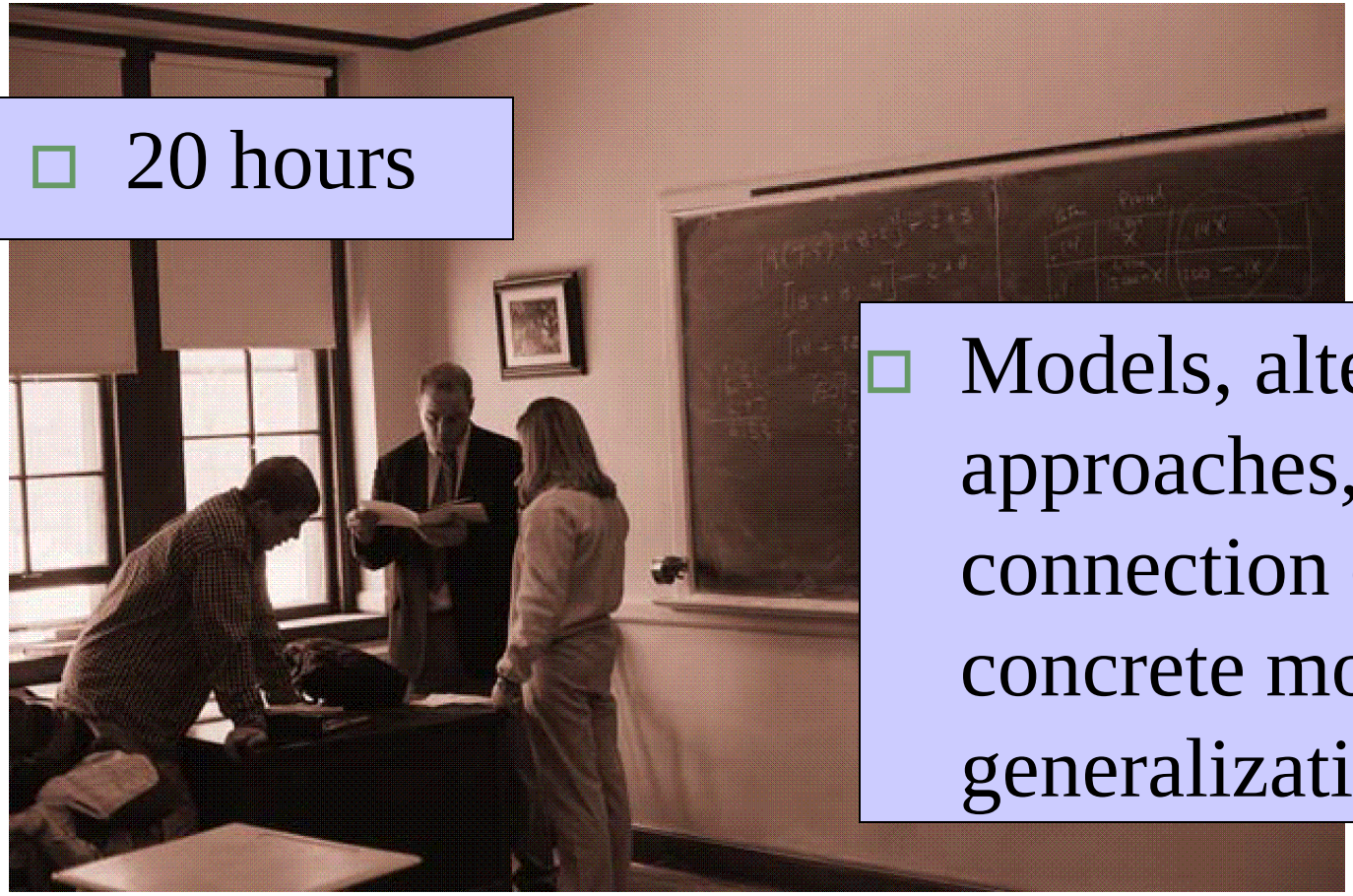
3. at least one regular math course but not teachers' course (63%)

4. teachers' course plus one or more other undergrad math (58%)

Teachers may need BOTH general math background and specialised background …

- The highest performing group in initial conceptual knowledge were those with at least one regular undergraduate math course PLUS the specialised undergraduate course for education students, but this is a small sample
- Candidates with math background but without specialised background initially did not start out as the strongest subgroup, but became so at the end
- Further data supports the idea that the more specialised experiences support the greatest growth

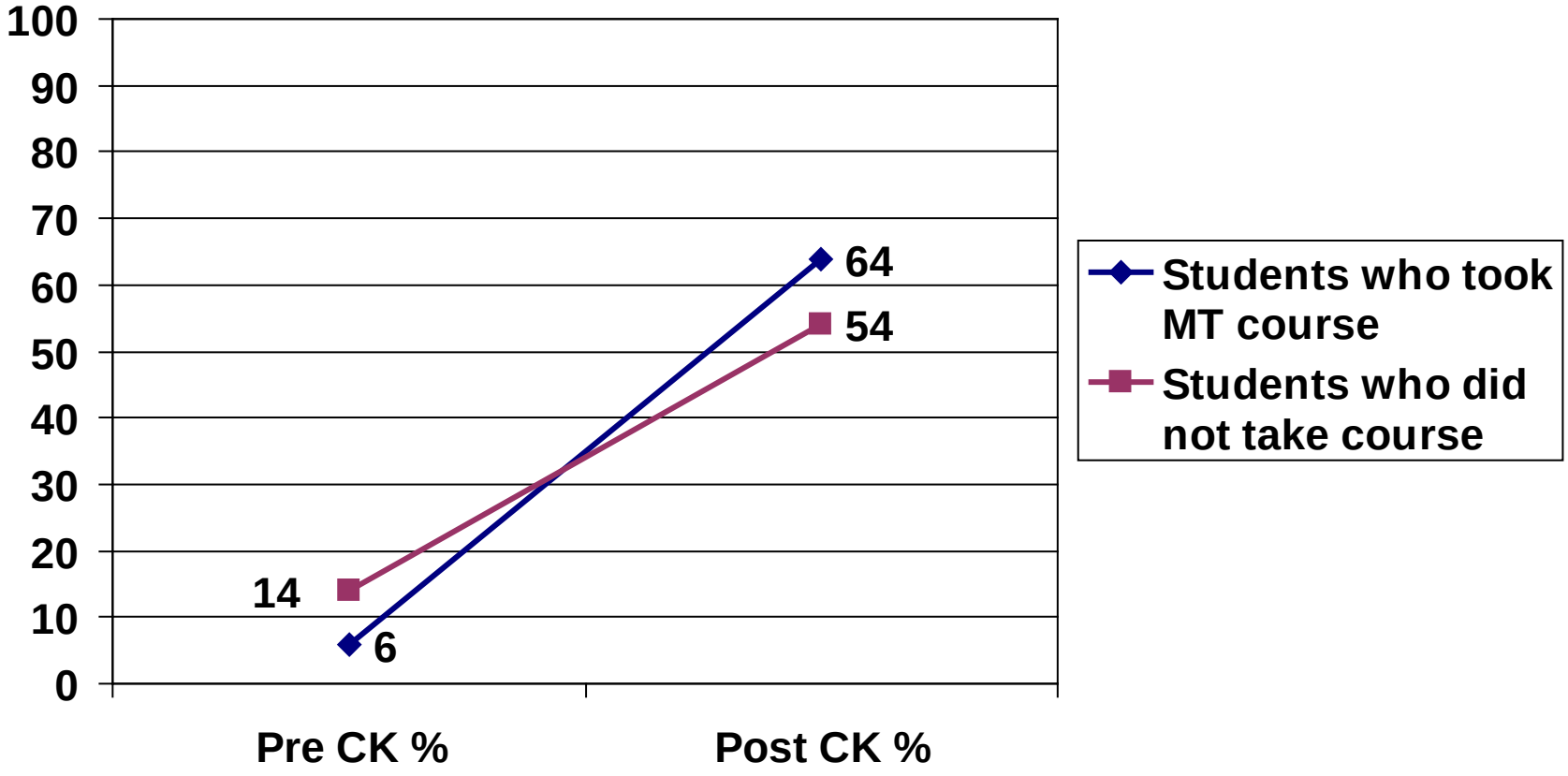
During the methods course …. *Math for Teaching* course



- 20 hours

- Models, alternate approaches, connection of concrete model to generalization

Conceptual Knowledge



2. Results: In-service teachers

- Professionally delivered mathematics in-service effectiveness examined (N=40)
- Professional Learning group meetings (about 45 meetings attended, 6 different teacher groups)
- Classroom observations and semi-structured interviews (N=4)
- Focus group meetings (6-8 teachers, 8 meetings)
- Survey results (N=50)
- *(Note that all samples are "biased")*

Results

- Professional development supports significant growth in conceptual understanding of mathematics, as well as corresponding beliefs changes
- Development is *relative* to initial position

Secondary vs. Elementary

- We have no evidence that secondary teachers generally have deeper conceptual understanding of intermediate mathematics; and we have some case study evidence that indicates they do not

Math as the “deal breaker” to reform:

- “I’m just no good in math. When I don’t get it the kids see that. I just have to go by the text because I don’t know what else to do. One of the biggest fears I have is will I teach it wrong or they will ask a question I do not have an answer to.”
- ….
- “Today we just had so much fun! When I get it I feel so confident and we can have so much fun exploring things in math. I wish I could do that all the time”

What “works” ?

- Individual mentorship
- Professional Learning groups with a strong task and goal-oriented focus and committed participants, who have access to a subject specialist as needed
- Professionally-delivered in-service opportunities which have a strong conceptual mathematics basis, *for those that volunteer*

Summary

- High school and university math background does make some difference in conceptual understanding of mathematics as needed for teaching
- Specialised undergraduate mathematics courses also contribute, including when taken in conjunction with other undergraduate math courses
- Highly specialized math courses taken concurrently with methods courses appear to help significantly
- Many in-service teachers are also in need of specialised content-based support

Conclusions ….

All teachers, **including** those with stronger levels of general mathematics background, need specialised opportunities for mathematics professional development for teaching.